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Absolute Infinity – A Bridge Between Mathematics and Theology?

Christian Tapp

Contents

1 Introduction

Mathematicians as well as theologians talk about 'absolute infinity'. In mathematics, the class of all ordinals, the class of all cardinals, and the class of all sets are examples of proper classes: assuming that one of them is a set leads to inconsistency. Even if they cannot be taken to be proper objects of mathematics, they are entities that in one way or the other *belong* to mathematics. (To what other scientific discipline could they belong? Who speaks about them if not the mathematician?) And they are infinite. Georg Cantor called their kind of infinity 'absolute infinity'. Theology and philosophy, in turn, describe God as an absolute or absolutely infinite being. Some have concluded that there is an immediate connection between mathematical and theological or philosophical conceptions of absolute infinity. Cantor's aleph series was said to be 'steps to the throne of God', absolute infinity was taken as a 'metaphysischer Grenzbegriff' (Clayton with reference to Kant).[1] The question is therefore: Does absolute infinity really bridge the gap between mathematics and theology?

I take it for granted that there is such a gap. Mathematics and theology are neither identical nor do they speak about the same objects or use the same methods.[2] The mere fact that there are people like Cantor, who have

[1]See Philip Clayton: *Das Gottesproblem, Band 1: Gott und Unendlichkeit in der neuzeitlichen Metaphysik*, Paderborn: Schöningh 1996, especially Chapter 6.

[2]There are surely parallels like the one between axiomatized mathematics and some

to say something of importance for both areas, hardly changes that, for biographical relations alone do not make a bridge between mathematics and theology.[3] It is an interesting question whether there are *topics* that both mathematics and theology are dealing with. One of the most promising candidates is: infinity.

At first one might observe that mathematicians use the word 'infinite' to refer to the size of sets of points or functions or functors or numbers, while theologians refer to God by it, the infinite creator of the finite world. As God is neither a set nor any other kind of mathematical object, mathematics and theology are not simply referring to the same thing or property when they use the word 'infinite'. Mathematics is not calculating within the realm of God's essence; and theology cannot make use of set theory as a means of producing new names of the unnameable.[4]

This observation does not imply that there are no links between the mathematical and the theological senses of 'infinity'. It implies only that the links, if existing, are more complicated and less explicit than a shared object, property, or method would be. I think it is promising to try to find such explicit links. But one should avoid premature claims such as 'infinity' simply means the same in the mouth of a mathematician and in the mouth of a theologian. Bluntly identifying mathematical and theological references to infinity leads into a nebular of supposed but not actual understanding.[5]

In this paper I want to focus on the case of Georg Cantor and his use of the term 'absolute infinity'. While Cantor in general clearly distinguished between the actual infinity of set theory (the 'transfinite') and the actual

sorts of deductive theology. But these seem to be parallels of presentation, not of method; and they are parallels, not identity relations. – Ivor Grattan-Guinness offers an interesting classification of possible links between Christianity and mathematics : "Christianity and Mathematics: Kinds of Link, and the Rare Occurrences after 1750", in: *Physis* 37/2 (2000), 467–500.

[3]There are, for example, distinguished mathematicians who are very religious people, and there are theologians who are more than laymen in mathematics; there are even priests who are professors of mathematics. I do not want to focus on such biographical relations in this paper – even though they are surely worth to be studied.

For the case of the mathematician Cantor and his exchange of letters with theologians concerning the set theory and its philosophical foundations, see my *Kardinalität und Kardinäle: Wissenschaftshistorische Aufarbeitung der Korrespondenz zwischen Georg Cantor und katholischen Theologen seiner Zeit* (= Boethius, vol. 53), Stuttgart: Franz Steiner 2005.

[4]This is true at least as long as one does not 'localize' all of reality in the mind of God, as in pantheistic positions. Note, that this is *toto caelo* different from saying that God *knows* all truths.

[5]The title of the voluminous monograph by Ludwig Neidhart: *Unendlichkeit im Schnittpunkt von Mathematik und Theologie*, Göttingen: Cuvillier 2005, 'Infinity in the intersection of Mathematics and Theology', suggest that there is such an intersection; the book, however, is simply assuming that and not arguing for it.

infinity of God, he used the expression 'actually infinite' in both cases and claimed some connections between mathematics and theology. Moreover, Cantor is considered a mathematician with deep theological inclinations and a strong sense for the frameworking and intermediative activity of philosophy. In the literature, Cantor is portrayed as someone who claims strong connections between religious, metaphysical, and mathematical realities.

One of the best known statements from Cantor in this context is that he called his series of transfinite cardinal numbers 'steps to the throne of God'. Some have taken this statement as evidence for that the late Cantor was mentally ill and held strong and strange religious views.[6] What exactly were Cantor's claims? How did he conceive of the relation of mathematical objects and God? This is what I want to deal with in this paper.

2 Alephs and God

As to what concerns the famous statement about Alephs as steps to the throne of God, one has first to note that this is not a quotation of Cantor's. It is somebody else's reporting about Cantor's views. The earliest evidence I was able to find dates back to 1950 when German mathematician Gerhard Kowalewski reports it in his autobiography. After having presented the definition of the Alephs as *Mächtigkeiten* of the transfinite number classes, Kowalewski writes:

> ... these powers, the Cantorian alephs, were for Cantor something holy, in a certain sense the steps which led up to the throne of the infinite, to the throne of God.[7]

This is Kowalewski's assessment of Cantor's opinion. Is that assessment right? Was it Cantor's conviction that the mathematical study of infinities leads to God? – In the following I want to analyze what Cantor really said about absolute infinity, the alephs, and God.

3 Absolute infinity

Cantor used the word 'absolute', i.e., its German equivalent '*absolut*', in several different ways. Some of them do not pertain to our way of inquiry as they are not directly related to the infinite. For example: 'absolute value'

[6]Compare the old caricature of Cantor receiving set theory as sort of a private revelation from God in P. Thuillier: "Dieu, Cantor, et l'infini", in: *La Recherche* 84 (1977), 1110–1116.

[7]'*Diese Mächtigkeiten, die Cantorschen Alephs, waren für Cantor etwas Heiliges, gewissermaßen die Stufen, die zum Throne der Unendlichkeit, zum Throne Gottes emporführen.*' Gerhard Kowalewski: *Bestand und Wandel. Meine Lebenserinnerungen – zugleich ein Beitrag zur neueren Geschichte der Mathematik*, München: Oldenbourg 1950, 201; translation by Michael Hallett in his *Cantorian Set Theory and Limitation of Size*, Oxford: Clarendon 1984, 44.

(often) or 'follows with absolute necessity' (GA 300),[8] Kant's 'absolute time' (*Grundlagen*, GA 192), 'absolute reality of space and time' (Cantor's Ph.D. defense thesis in Latin, GA 31), 'absolute concept of power/cardinality' ('*absoluter Mächtigkeitsbegriff*', that is, without relativization to continuity as in Steiner's works where Cantor found the notion of cardinality, see *Über unendliche, lineare*, GA 151). If we sort out these usages of the word 'absolute' the remaining ones have something to do with infinity. In the following, I will try to spell out what they mean.

The main sense in which Cantor uses the predicate 'absolutely infinite' is that of what we today call 'proper classes'. He calls *On*, the class of all ordinal numbers, as well as *Card*, the class of all cardinal numbers, 'absolutely infinite'. This usage can be found all over his works, but most prominently in his letters to Dedekind from around 1900.[9] In these letters, Cantor used the expression 'absolutely infinite' synonymously with 'inconsistent' in order to denote inconsistent multiplicities that cannot be conceived of as sets, i.e., for proper classes.[10] His solution to the so-called 'antinomies of set theory' was simply to take these arguments as usual indirect proofs: the contradictions derived were disproving the assumption that *On* or *Card* or the class of all sets were sets themselves. Insted, they are absolutely infinite multitudes (*Vielheiten*). They are 'too large' to be tractable as genuine objects of set theory.[11]

This is the "technical sense" of 'absolute infinity' in Cantor's set theory. First, however, Cantor used the expression 'absolutely infinite' in a non-technical sense, for example when he called the 'aggregate' (*Inbegriff*)[12] of the integers 'absolutely infinite'. He did so already in his papers on trigonometric series in 1872, that is, at a time at which even his most charitable

[8]Most of Cantor's works are accessible via the collection Georg Cantor: *Gesammelte Abhandlungen mathemathischen und philosophischen Inhalts*, ed. Ernst Zermelo, Berlin: Springer 1932, reprint Hildesheim: Olms ^{2}1962, henceforth cited as 'GA'. For a complete list of Cantor's publications see my *Kardinalität und Kardinäle* (note 3), 578–582.

[9]To be more precise, Cantor used the predicate 'absolutely infinite' as a predicate for concepts (see GA 95), sequences (*Grundlagen*: GA 167, 195, 205), aggregates (*Inbegriffe*; *Grundlagen*: GA 205), and multiplicities (*Vielheiten*; Letter to Dedekind, 28.7.1899: GA 445).

[10]Cf. Cantor to Dedekind: '*Das System Ω aller Zahlen ist eine inconsistente, eine absolut unendliche Vielheit.*' Letter dated 3.8.1899 (in GA 445 errorneously edited within a letter dated 28.7.1899; quoted from Georg Cantor: *Briefe*, ed. Herbert Meschkowski and Winfried Nilson, Berlin: Springer 1991, 408).

[11]For the idea of set theory as a theory of objects with *limited* size see Michael Hallett: *Cantorian Set Theory and Limitation of Size*, Oxford: Clarendon 1984.

[12]The translation of '*Inbegriff*' as 'aggregate' is due to William B. Ewald and his translation of Cantor's *Grundlagen*, see William B. Ewald: *From Kant to Hilbert: A source book in the foundations of mathematics*, Vol. 2. Oxford: Clarendon, for example p. 916.

interpreters do not ascribe to him a clear vision of the 'antinomies'.

It is much harder to say what exactly Cantor meant by 'absolutely infinite' during the intermediate period between the early papers on trigonometric series and the late correspondence with Dedekind.

In *Über die verschiedenen Standpunkte* of 1885/6, Cantor differentiates between transfinite and absolute infinity. The difference is that transfinite infinity can still be augmented while absolute infinity can not.[13] In the *Mitteilungen* of 1887/8, Cantor gives a definition of 'actual infinity' as a quantity of a size exceeding all finite sizes of the same kind.[14] He sticks to the distinction of transfinite and absolute infinity as two kinds of actual infinity calling them shortly '*Transfinitum*' and '*Absolutum*'. Hence, Cantor's conception of 'the absolute' in these context is a quantitative concept. As such, it must not be confused with the absolute of, say, idealist philosophy. In Cantor's understanding, the quantitative absolute differs from the transfinite only in that it is non-augmentable.[15]

Cantor does not say what exactly he has in mind when he talks about 'augmentability'. For sure, the absolutely infinite class of transfinite cardinal numbers can in a sense be augmented by adding some ordinals to it which are not (identifiable to) cardinals. Hence, on the one hand, there is a sense in which absolutely infinite multiplicities can be augmented. On the other hand, however, it would be too restrictive to call a multiplicity 'augmentable' only if there are further objects of the same kind as the object of the multiplicity; for then the set of finite integers – the prime example for transfinite infinity – would be absolutely infinite (trivially there are no finite integers in addition to the set of finite integers). Concludingly, it is not completely clear what Cantor meant by his definition of absolute infinity in terms of non-augmentability.

Some light may be brought to this problem by taking into account that Cantor has studied a book on natural philosophy by the 19$^{\text{th}}$ century Jesuit father Tilmann Pesch. Pesch defined infinity as '*id, quo non sit maius, nec esse possit*', 'that than which there is nothing bigger or could be'.[16] Cantor saw clearly that this definition is inadequate as it defines a maximum which does not need to be infinite. Despite this criticism, it may be the case that Pesch's definition lead Cantor to take non-augmentability as the characteristic property of absolute infinity.

As considered in itself, this usage of 'absolutely infinite' is completely unproblematic. But problems arise when it is put into an intimate relation

[13] *Über die verschiedenen Standpunkte*, GA 375.
[14] GA 401.
[15] GA 394, 401, passim.
[16] Tilmann Pesch: *Institutiones philosophiae naturalis: Secundum principia S. Thomas Aquinatis ad usum scholasticum*, Freiburg: Herder 1883, § 403.

to a philosophical concept of infinity as Kant has in his antinomies of pure reason. Cantor thought Kant's antinomies to be flawed in not distinguishing finely enough between different kinds of infinity.[17] Cantor is surely touching a huge philosophical problem when he considers some post-Kantian philosophers to be misguided in conceiving the absolute as the ideal borderline of the finite. Unfortunately, Cantor does not present his thoughts about this problem in any greater detail.

In *Über die verschiedenen Standpunkte*, Cantor uses a threefold distinction in order to classify the positions of other philosophers, theologians, and mathematicians with respect to the reality of the actually infinite. He distinguishes between the infinite *in Deo*, *in mundo*, and *in abstracto*.[18]

In the beginning of the *Mitteilungen*,[19] Cantor combines this distinction and the absolute/transfinite distinction to the following matrix:

actual infinity	in Deo	in mundo	in abstracto
augmentable		the transfinite → *Metaphysics*	transfinite numbers → *Mathematics*
non-augmentable	the absolute [1886], absolute infinity [1887][20] → *Theology*		

The blank fields in this table are really empty: neither does Cantor say that there is only augmentable infinity in nature, nor does he say there is non-augmentable infinity in nature. In the *Mitteilungen*, Cantor's point is only to make a sharp distinction between transfinite and absolute infinity and to use this distinction in order to separate the disciplines of mathematics, metaphysics, and theology.

4 Cantor on divine attributes

Cantor mentions several divine attributes in the *Mitteilungen* (1887/8): absolute freedom (GA 387,400), absolute omnipotence (GA 396), absolutely inscrutable power of the will (GA 404), absolute intelligence (GA 401,402),

[17] *Über die verschiedenen Standpunkte*, GA 375; cp. Zermelo's discussion in note [1], GA 377.
[18] GA 372. The expressions '*in Deo*' and '*in abstracto*' are used by Cantor, the expression '*in mundo*' is added by me in order to have a handy concept available.
[19] GA 378.
[20] In *Über die verschiedenen Standpunkte* [1885/6], Cantor says 'the Absolute' (GA 372); in the *Mitteilungen* [1887/8], he says 'absolute infinity or, shortly, the absolute' (GA 378). So, at the time of the *Mitteilungen*, he surely used 'absolute infinity' and 'the absolute' synonymously.

and the capability of absolutely free decisions (GA 406).[21] God and all of his attributes are '*infinitum aeternum increatum sive Absolutum*' – the eternal, uncreated infinite or absolute (GA 399). In these considerations, the term 'absolute' has no explicit quantitative connotation, nor is it explicitly related to proper classes as in the letters to Dedekind. Here, 'the absolute' is used in a philosophical or theological sense as derived from the literal meaning of the Latin '*absolutum*', stemming from '*absolvere*' – to be detached or disassociated. Using natural language predicates as predicates for the unrestrained God requires detaching or disassociating them from limitations of their natural meanings.

5 A methodological parallel

There is an interesting methodological parallel between Cantor's mathematical theory and traditional theology. When Cantor analyzed traditional proofs of the impossibility of infinite numbers with the intention to disprove them, he found that the most common mistake in them was to transfer propositions holding in the domain of the finite without further qualification to the domain of the infinite. This is what he called the '*proton pseudos*', the cardinal error of those anti-infinity arguments. One may formulate this *proton pseudos* positively, as a methodological maxim: do not carelessly transfer insights from the finite domain to the domain of the infinite.

In this form, this maxim plays a major role in the traditional theological doctrine of God. In religious speech, we cannot help but use the vocabulary of our everyday language that acquired its meaning by being used in our everyday life. But in order to use this vocabulary for God, who is not an ordinary object of our everyday language, it has to run through kind of a purgatorial process traditionally called *via positiva*, *via negativa*, and *via eminentiae*. To put it shortly and a little laxly: part of the positive content of our concepts must be kept, the negative content of limitations, the creaturely mode of being, must be crossed out, and their meanings must be 'boosted' from the limited world of creatures to the unlimited and uncreated creator. Albeit I cannot go into the details of this maxim of theological semantics, the methodological parallel should be clear: no simple transfer of propositions or meanings from the realm of the finite to the realm of the infinite – be it the realm of infinite numbers or an infinite God.

6 Mathematical infinity and God

Cantor sometimes explicitly speaks about relations between the actual infinite of mathematics and God. The first place where he does so is in *Über

[21] German original: '*absolute Freiheit*', '*absolute Omnipotenz*', '*absolut unermeßliche Willenskraft*', '*absolute Intelligenz*', and '*absolut freier Ratschluß*'.

die verschiedenen Standpunkte. I quote from the German text and comment on it afterwards as it is hardly translatable:

> *Wenn aber aus einer berechtigten Abneigung gegen solches illegitime A. U. sich in breiten Schichten der Wissenschaft, unter dem Einflusse der modernen epikureisch-materialistischen Zeitrichtung, ein gewisser Horror Infiniti ausgebildet hat, der in dem erwähnten Schreiben von Gauß seinen klassischen Ausdruck und Rückhalt gefunden, so scheint mir die damit verbundene unkritische Ablehnung des legitimen A. U. kein geringeres Vergehen wider die Natur der Dinge zu sein, die man zu nehmen hat, wie sie sind, und es läßt sich dieses Verhalten auch als eine Art Kurzsichtigkeit auffassen, welche die Möglichkeit raubt, das A. U. zu sehen, obwohl es in seinem höchsten, absoluten Träger uns geschaffen hat und erhält und in seinen sekundären, transfiniten Formen und allüberall umgibt und sogar unserm Geiste selbst innewohnt.*[22]

The key point of this one German sentence may be paraphrased like this: In the time of Gauss there was a *Horror infiniti* in mathematics that led to rejecting not only illegitimate notions of infinity but also legitimate ones. But rejecting the legitimate forms of mathematical infinity results in a certain short-sightedness: one becomes incapable of seeing the 'highest, absolute bearer' of absolute infinity, namely God, the creator of the world. When Cantor says that depriving mathematics of actually infinite numbers means to lose a cognition of God, he indeed seems to cross a bridge between mathematics and theology.

But what is the relation between actually infinite numbers and God? According to Cantor, God is the highest, absolute bearer of actual infinity, while the secondary, transfinite forms of infinity are all around us and even in our minds. I interpret this 'forms of infinity all around us' as the *transfinitum in mundo*, and the belief in their existence as the belief that there are infinite sets of entities in our universe.[23] And I take 'being inherent to our minds' as alluding to the nature of infinite numbers as abstract objects, '*Zusammenfassungen zu einem Ganzen*', 'collections into a whole', according to Cantor's famous definition.[24] Calling both, the infinite sets of natural objects and the infinite numbers 'secondary forms' might be a hint at Spinoza, whose works Cantor has studied in some detail (especially the

[22] GA 374–375.

[23] For this claim (that there are infinitely many entities in our universe) to be true, it may be necessary not to restrict 'entities' to '*wirkliche*' entities in Bolzano's sense. If one admits sentences in themselves (*Sätze an sich*) or propositions or facts or whatever the like as constituents of the world, one can be sure that there are infinitely many of them 'in the world'. That does, however, not mean that one is committed to infinitely many real things in the world, where 'real' is taken in the strong, Bolzanoean sense.

[24] See *Beiträge* I (1895), GA 282; English translation of the *Beiträge* by P. E. Jourdain as *Contributions to the founding of the theory of transfinite numbers*, New York: Dover 1915.

Ethics).[25]

A second example for direct relations between mathematical and theological subjects can be found in Cantor's letters to theologians. In one of these letters Cantor interprets a proposition from the dogmatic consitution '*Dei filius*' of the first Vatican council. This proposition says about God that he is

> inexpressibly loftier than anything besides himself which either exists or can be imagined[26]

Cantor adds that God's ineffability would be the more considerable the more extended the area of things below him is. In this sense he writes in a letter to the Dominican father Thomas Esser:

> Every extension of our insight into what is possible in creation leads necessarily to an extended cognition of God.[27]

Cantor's reasoning is thus: The more cardinalities we have the more sets of things are possible, and the more sets of things are possible the more circumstances in nature can be expressed, and the more circumstances in nature can be expressed the greater for us is a God who is, in a sense, 'above' nature.

There are only very few passages in Cantor's works that suggest such an immediate link between theological and mathematical subjects. Other passages suggest more caution. So, for example, in the *Grundlagen*, Cantor says that

> the true infinite or Absolute, which is in God, permits no determination whatsoever[28]

While transfinite sets and numbers are perfectly determined or rather determinable,[29] God's infinity cannot be determined. Hence Cantor clearly draws a line between the infinity of sets and the infinity of God. What then

[25]See Paolo Bussotti and Christian Tapp: "The influence of Spinoza's concept of infinity on Cantor's set theory", In: *Studies in History and Philosophy of Science* 40 (2009), 25–35.

[26]'*Super omnia, quae praeter ipsum sunt et concipi possunt, ineffabiliter excelsus*', see: Denzinger, Heinrich / Hünermann, Peter: *Kompendium der Glaubensbekenntnisse und kirchlichen Lehrentscheidungen / Enchiridion symbolorum et definitionum ...*, Freiburg: Herder [40]2005, no. 3001; transl. Norman Tanner (ed.): *Decrees of the Ecumenical Councils*, Vol. 2: Trent to Vatican II, London: Sheed & Ward 1990, 805.

[27]'*Jede Erweiterung unserer Einsicht in das Gebiet des Creatürlich-möglichen muß daher zu einer erweiterten Gotteserkenntnis führen.*' See my *Kardinalität und Kardinäle* (note 3), letter [CanEss96], p. 308, and the commentary on p. 86.

[28]'*Daß das wahre Unendliche oder Absolute, welches in Gott ist, keinerlei Determination gestattet*', *Grundlagen* § 5, GA 175; transl. Ewald: *From Kant to Frege* (note 12), 891.

[29]In the *Mitteilungen*, there is a similar passage about the Absolute (GA 405-406):

about proper classes? Are they undetermined in this sense? This question is central now, for if proper classes are determined, this would show a gap between mathematical absolute infinity and God's absolute infinity.

To my knowledge Cantor never says that proper classes or inconsistent multiplicities are undetermined. Rather, the examples of classes or inconsistent multiplicities Cantor discusses all fulfil one of the criteria for sets, namely that for every object it must be determined whether it belongs to the collection or not. (The difference between sets and classes lies in the fact that thinking of the elements of a proper class to be together in forming a new object, a set, leads to a contradiction, and not in that the elementhood relation would be vague in any sense.) If this kind of determination is meant in the quotation above, then one has to conclude that the difference mentioned by Cantor is not in the first place a difference between the transfinitely and the absolutely infinite mathematical objects, but a much more general difference between something determined (that only can be subject of mathematics) and something undetermined.

The most important passage in Cantor's writings is a passage from his endnotes to § 4 of the *Grundlagen*. It is almost always cited if something about Cantor's views on the relation between absolutely infinite mathematical objects and the absolute infinity of God is at issue. The two best known quotations are:

> The absolute can only be acknowledged [[*anerkannt werden*]] but never known [[*erkannt werden*]][30]

and

> the absolutely infinite sequence of numbers thus seems to me to be an appropriate symbol of the absolute.

> *Das Transfinite [...] weist mit Notwendigkeit auf ein Absolutes hin, auf das 'wahrhaft Unendliche', an dessen Größe keinerlei Hinzufügung oder Abnahme statthaben kann und welches daher quantitativ als absolutes Maximum anzusehen ist. Letzteres übersteigt gewissermaßen die menschliche Fassungskraft und entzieht sich namentlich mathematischer Determination.*

> The transfinite [...] points with necessity to an Absolute, to the 'truly infinite', whose magnitude can neither be augmented nor diminished and which is, hence, quantitatively to be seen as an absolute maximum. In a sense, it transcends human cognitive powers and withstands mathematical determination in particular. [my transl.]

This quotation does not really help clarifying the relation between the mathematical and the metaphysical absolute. In the context of the *Mitteilungen*, 'the absolute' is quasi-defined as an inaugmentable actual infinite. It is not clear whether Cantor wants it to refer to a quantitative concept, to a general metaphysical concept, or to both. In my view, this passage from the *Mitteilungen* is compatible with either interpretation and, hence, does not help settling our question.

[30]'*Werden*' added to the German insertions, C. T.

In order to grasp the exact sense of what Cantor says in these quotations we need to consider their full context. The whole passage is, however, a little dark and difficult to understand:

> I have no doubt that, as we pursue this path [of investigating transfinite numbers not only in mathematics but wherever they may occur, C. T.] ever further, we shall never reach a boundary that cannot be crossed; but that we shall also never achieve even an approximate conception of the absolute. The absolute can only be acknowledged [[*anerkannt*]] but never known [[*erkannt*]] – and not even approximately known. For just as in number-class (I) every finite number, however great, always has the same power of finite numbers greater than it, so every suprafinite number, however great, of any of the higher number-classes (II) or (III), etc. is followed by an aggregate of numbers and number-classes whose power is not in the slightest reduced compared to the entire absolutely infinite aggregate of numbers, starting with 1. As Albrecht von Haller says of eternity: 'I attain to the enormous number, but you, o eternity, lie always ahead of me.' The absolutely infinite sequence of numbers thus seems to me to be an appropriate symbol of the absolute; in contrast, the infinity of the first number-class (I), which has hitherto sufficed, because I consider it to be a graspable idea (not a representation [[Vorstellung]]), seems to me to dwindle into nothingness by comparison.[31]

Let me start my analysis with the less problematic point that 'the absolute can only be acknowledged [[*anerkannt*]] but never known [[*erkannt*]]'. The question is first, whether 'the absolute' refers to inconsistent multiplicities or to God. The problem is that the preceding sentence suggests that Cantor is talking about the absolute in the sense of God, while the following sentence suggests the opposite.

In the preceding sentence Cantor says that 'we shall never achieve even an approximate conception of the absolute'[32] by all our investigations in

[31]GA 205; translation in Ewald, *From Kant to Hilbert*, 916. The German original reads:

> *Daß wir auf diesem Wege [die transfiniten Zahlen nicht nur mathematisch, sondern überall, wo sie vorkommen, zu untersuchen, C. T.] immer weiter, niemals an eine unübersteigbare Grenze, aber auch zu keinem auch nur angenäherten Erfassen des Absoluten gelangen werden, unterliegt für mich keinem Zweifel. Das Absolute kann nur anerkannt, aber nie erkannt, auch nicht annähernd erkannt werden. Denn wie man innerhalb der ersten Zahlenklasse (I) bei jeder noch so großen endlichen Zahl immer dieselbe Mächtigkeit der ihr größeren endlichen Zahlen vor sich hat, ebenso folgt auf jede noch so große überendliche Zahl irgendeiner der höheren Zahlenklassen (II) oder (III) usw. ein Inbegriff von Zahlen und Zahlenklassen, der an Mächtigkeit nicht das mindeste eingebüßt hat gegen das Ganze des von 1 anfangenden absolut unendlichen Zahleninbegriffs. Es verhält sich damit ähnlich, wie Albrecht von Haller von der Ewigkeit sagt: 'ich zieh' sie ab (die ungeheure Zahl) und Du (die Ewigkeit) liegst ganz vor mir.' Die absolut unendliche Zahlenfolge erscheint mir daher in gewissem Sinne als ein geeignetes Symbol des Absoluten; wogegen die Unendlichkeit der ersten Zahlenklasse (I), welche bisher dazu allein gedient hat, mir, eben weil ich sie für eine faßbare Idee (nicht Vorstellung) halte, wie ein ganz verschwindendes Nichts im Vergleich mit jener vorkommt.*

[32]'*Auch nur angenäherten Erfassen des Absoluten,*' GA 205.

transfinite numbers and sets. If even an *approximate* conception of the absolute in question is denied, only the absolute infinity of God can be at issue. For Cantor would probably say that it is possible to gain approximative knowledge about proper classes like the class of all ordinal numbers at least in the sense in which one also has approximative knowledge about a concept by learning more and more about things falling under it. Acquisition of this kind of knowledge is also possible in case of proper classes, as Cantor would probably admit. Hence, the preceding sentence suggests to read 'the absolute' in the sense of 'God'.

The beginning of the following sentence raises some doubts, however. The sentence begins with 'for' suggesting that it will present a reason for the earlier claim. But now Cantor is explicitly talking about number classes. So either this passage is obscure, or Cantor feels entitled to this transition between the two senses of 'the absolute' because he is convinced that there are in fact relations between them allowing for that transition. We will come back to this point shortly.

The second quotation reads:

> the absolutely infinite sequence of numbers thus seems to me to be an appropriate symbol of the absolute.[33]

It is obvious but sometimes overlooked that according to this statement the absolutely infinite series of ordinal numbers is *not* said to be the absolute or to lead to the absolute (like steps lead to a throne), but to be a *symbol* of the absolute. One might express this in terms of the metaphor of steps to the throne of God, saying that numbers are steps to His throne, not to God himself. But the sentence does not stop here:

> The absolutely infinite sequence of numbers thus seems to me to be an appropriate symbol of the absolute; in contrast, the infinity of the first number-class (I), which has hitherto sufficed, because I consider it to be a graspable idea (not a representation [[*Vorstellung*]]), seems to me to dwindle into nothingness by comparison.

What Cantor has in mind here is the function or role the sequence of finite natural numbers has played in the history of theology and metaphysics. It was used as an image or a symbol for God's infinity. This role is now to be taken by the whole, absolutely infinite series of all ordinal numbers for the following reason: In the light of Cantor's theory of transfinite numbers, the series of natural numbers turns out to be intrinsically limited, namely by the ordinal number ω which is the smallest transfinite ordinal number, but greater than every natural number. It is very plausible that Cantor found such an intrinsic limitation inappropriate for a symbol of the absolute. The

[33]'*Die absolut unendliche Zahlenfolge erscheint mir daher in gewissem Sinne als ein geeignetes Symbol des Absoluten,*' GA 205.

whole sequence of transfinite ordinal numbers, in contrast, does not suffer this limitation. The fact that the class of all ordinal numbers is not a set but a proper class, makes it more appropriate as a symbol for the unlimited God than the limited series of natural numbers. To put it in a nutshell: due to its mathematical absoluteness *On* is a better symbol of the Absolute than the limited ω.

Cantor grasps this difference between ω and *On* also in terms of calling ω a 'graspable idea'. Tacitly that means that *On* is not a graspable idea. This once more confirms the thesis that already at the time of the *Grundlagen* Cantor had a perfectly clear 'solution' to the paradoxes, or better: was well aware of the arguments later used in the paradoxes. For him, these arguments were simply reductions to the absurd of the assumption that *On* was a set or a 'graspable idea'.

This interpretation of Cantor's second famous statement about the absolute suggests a solution to a problem which remained open in interpreting Cantor's first statement about the incognizability of the absolute. One may read his statements as reasonings *via the symbolization*: the absolutely infinite series is a better symbol for the absolute infinity of God than the series of natural numbers for it is not limited in the same way (by an ordinal number like ω). This symbol is convenient as one cannot climb to an epistemic position that allows one to scrutinize what it symbolizes 'from the top': there is no top above God, and there is no top above the series of ordinals. One cannot grasp the whole absolutely infinite series of transfinite ordinal numbers at once as a whole, it does not form 'a graspable idea'. Subtracting finite amounts (and even infinite amounts) does not diminish the magnitude of the class of ordinals as it does not diminish the magnitude of God. Cantor's point in the first statement is that his research in the realm of the infinite – the mathematical research about transfinite ordinal numbers as well as the metaphysical research about transfinite sets of natural objects – does *not* lead to any direct knowledge of God. Although one can 'manage' higher and higher ordinal systems (just think about the ordinal notation systems in proof theory, or the huge cardinals in contemporary set theory) with no intrinsic limit with respect to the size of the ordinals thus considered, one will not arrive at a direct cognition of the absolutely infinite multiplicity of all ordinal numbers, and *a fortiori* not at a direct cognition of God. As the class of ordinals cannot be fully embraced by mathematical thinking, so God cannot by theological thinking.

> I have no doubt that, as we pursue this path ever further, we shall never reach a boundary that cannot be crossed; but that we shall also never achieve even an approximate conception of the absolute.[34]

[34]'*Daß wir auf diesem Wege immer weiter, niemals an eine unübersteigbare Grenze,*

This does not mean that no form of knowledge about God or no cognition of God is possible. Cantor says only that his research in the realm of the transfinite does not lead to such a direct knowledge or cognition.[35]

As to what concerns absolute infinity as a possible bridge between mathematics and theology, my thesis is therefore: as Cantor has show us, there are methodological parallels between set theory and theological semantics, and there is a relation between absolute infinity of God and the absolute infinity of the series of transfinite ordinal numbers. But this relation is a symbolic one: set theory does not produce direct knowledge of God.[36]

aber auch zu keinem auch nur angenäherten Erfassen des Absoluten gelangen werden, unterliegt für mich keinem Zweifel.'

[35]The English translation is a little bit misleading here. From the German original it is clear that 'as we pursue this path' must be read as qualification not only for 'never reach a boundary' but also for 'never achieve even an approximate conception of the absolute.'

[36]An earlier version of this paper was presented during the conference honoring Harvey Friedman, May 15, 2009. I am indebted to Leon Horsten and James Bradley for their helpful comments.